1. **Problem:** Let X be a metrizable TVS and Y a TVS. Let $T : X \to Y$ be a linear map such that for every sequence $\{x_n\}_{n\geq 1} \subset X, x_n \to 0$ implies, $\{T(x_n)\}_{n\geq 1}$ is a Cauchy sequence. Show that T is continuous.

   Solution: See, Thm 1.32 in the book "Functional Analysis" by W. Rudin.

2. **Problem:** Consider $\Lambda_m : l^2 \to \mathbb{C}$ defined by $\Lambda_m(x) = \sum_1^m n^2 x(n)$. Let $x_n = \frac{1}{n} e_n$. Show that $K = \{x_n\} \cup \{0\}$ is compact. Show that each $\Lambda_m(K)$ is a bounded set but $\{\Lambda_m(K)\}_{m\geq 1}$ is not uniformly bounded.

   Solution: Let $\{V_\alpha\}$ is an arbitrary open cover of K so that 0 is contained in an open set say V_α. Since $x_n = \frac{1}{n} e_n \to 0$ in l_2, we shall have $x_n \in V_\alpha$ except finitely many points and those finitely many x_n can be covered in finitely many open sets.
   Therefore, K can be covered by finitely many open sets and hence it is compact.

   For the other part note that

   $$\begin{aligned} \Lambda_m(x_n) \quad &= m \ \text{ if } \ n \leq m \\ &= 0 \ \text{ if } \ n > m. \end{aligned}$$

   This shows that $\Lambda_m(K)$ is a bounded set but $\{\Lambda_m(K)\}_{m\geq 1}$ is not uniformly bounded.

3. **Problem:** Let X be a separable normed linear space. Show that X^*, with the weak*- topology is separable.

   Solution: See Lemma 10.15. from the book "Linear Chaos" by Karl-G. Grosse-Erdmann and Alfred Peris Manguillot.

4. **Problem:** Let X be a $LCTVS$ and A, B two compact convex sets. Show that the extreme points of the convex hull, $\partial_e CO(A \cup B) = \partial_e A \cup \partial_e B$.

   Solution: It may happen that $\partial_e A \cup \partial_e B \nsubseteq \partial_e CO(A \cup B)$. I am giving an example bellow:

   Take $X = \mathbb{R}, A = [1, 2], B = [2, 3]$. Then we get $CO(A \cup B) = [1, 3]$ and $\partial_e CO(A \cup B) = \{1, 3\}$ where $\partial_e A \cup \partial_e B = \{1, 2, 3\} \nsubseteq \partial_e CO(A \cup B)$.

5. **Problem:** Let K be a convex, compact set in a LCTVS X. Let $a : K \to [0, 1]$ be an onto, affine, continuous map. Show that $a^{-1}\{0\}$ is an extreme convex set.

   Solution: The set $S := a^{-1}\{0\}$ is non-empty as it is given that a is onto.
   First we will show that S is a convext set.
   Take $x, y \in S$ and $\lambda \in (0, 1)$.
   Now, using the fact that a is affine map and $ax = ay = 0$ we get
   $a((1 - \lambda)x + \lambda y) = (1 - \lambda)ax + \lambda ay = 0$ which shows that S is a convex set.
   We now show that S is an extreme set.
   Take $z \in S$ such that $z = (1 - \lambda)x + \lambda y$ for some $x, y \in K$.
   Applying a both sides we get, $az = (1 - \lambda)ax + \lambda ay$.
   Now, as $z \in S$ we have $(1 - \lambda)ax + \lambda ay = 0$.
   But as $\lambda \geq 0, (1 - \lambda) \geq 0$ and $ax \geq 0, ay \geq 0$ the above equality is true only when $ax = ay = 0$ i.e $x, y \in S$.
   Hence, we are done.

6. **Problem:** Let K be a compact, convext set in a LCTVS. Let $x_0 \in K$. Show that there exists a probability measure μ with $\mu(\overline{\partial_e K}) = 1$ and the resultant $\gamma(\mu) = x_0$.

   Solution: For the solution one can see the paper of E. Bishop, K. de Leeuw, "The representations of linear fun tionals by measures on set of extreme points", Ann. Inst. Fourier (Grenoble), 9 (1959), 305-331.

   In this paper they proved Choquet's theorem without the assumption that K is metrizable.

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9. **Problem:** Give complete details to show that any closed subspace of l^1 has the Radon-Nikodym property.

10. **Problem:** State and prove tha Exhaustion Lemma for vector measures.

    Solution: See Lemma:4 from the book "VECTOR MEASURES" written by J. DIESTEL and J.J. UHL, JR.